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Uniform Fatou's Lemma

Abstract

Fatou's lemma is a classic fact stating that the limit inferior of expectations is greater than or equal to the expectation of the inferior limit. This talk introduces a stronger inequality that holds uniformly for integrals on measurable subsets of a measurable space. The necessary and sufficient condition, under which this inequality holds for a sequence of probabilities converging in total variation, is provided. This statement is called the uniform Fatou lemma, and it holds under the minor assumption that all the expectations are well-defined. The uniform Fatou lemma improves the classic Fatou lemma in the following directions: the uniform Fatou lemma states a more precise inequality, it provides the necessary and sufficient condition, and it deals with variable probabilities. As one of applications, the uniform Fatou's lemma can be used to prove continuity of transition probabilities for Partially Observable Markov Decision Processes.

The talk is based on joint papers with Pavlo O. Kasyanov and Michael Z. Zgurovsky